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Theory of πK scattering near threshold

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Abstract. A simple theory is developed for the process $\pi K \rightarrow \pi K$ near threshold. It is similar to one recently proposed for $\pi\pi \rightarrow \pi\pi$, and rests on an assumption of smoothness of the scattering amplitudes as functions of the energy–momentum invariants. The Adler PCAC consistency condition is imposed, but instead of current commutators, rho-meson dominance on the mass shell is used to fix overall normalization. The two approaches are connected by the KSRF relation, but because this model depends on two parameters (mass, width) more information is obtained. Thus the model contains P and D waves, and gives improved scattering-length predictions. Comparison with results from crossing-symmetric Regge models suggests that these need important additional terms.

1. Introduction

Now that detailed experimental studies of πK scattering are becoming possible (Kane 1970, Schlein 1970), there is growing interest in related theoretical predictions†. The approach which is developed here applies to the πK interaction near threshold, and is similar to one recently proposed for the process $\pi\pi \rightarrow \pi\pi$ (Graham and Johnson 1969).

The basic assumption is that the appropriate scattering amplitudes are smooth and simple functions of the energy–momentum invariants on and near the pion mass shell. As explained in connection with the $\pi\pi$ process, this attractive hypothesis stems from the unproven but plausible idea that physical quantities are as free as is possible (consistent with general principles) from singularities and pathological variations. In particular, although unitarity compels the existence of threshold branch points we suppose in the absence of contrary information that they are relatively weak. In fact, with the knowledge that there are no πK bound states or light resonances, we assume that it is a good first approximation to take the low-energy amplitudes to be purely real. Then, of course, the model is consistent only if it predicts small scattering lengths.

In the next section, the πK isospin amplitudes are constructed from suitably crossing-symmetric polynomials in the usual s, t, u variables. Terms up to quadratic are included, and the absence of prominent isospin-3/2 resonances is incorporated in a conventional way. The polynomials are chosen so that the amplitudes obey the Adler (1965) consistency condition; that is they are constrained to vanish at values of s, t, u corresponding to the mass of either pion being taken to zero. This condition leaves only two undetermined constants in the amplitudes, and an on-mass-shell ρ-dominance assumption fixes these in terms of the resonance parameters, assumed given. Universality of the rho meson coupling may then be used to relate $\rho K\bar{K}$ and $\rho\pi\pi$ vertices and make numerical predictions.

From the on-shell amplitudes, scattering lengths and threshold cross-sections can be read off. They turn out to be small (thus confirming the consistency of the

† This was also discussed at the 1969 Argonne–Purdue Conference on $\pi\pi$ and Kπ interactions. (The proceedings have not been published.)

model), and similar to the well known results of Weinberg (1966). However the present model contains important improvements over the old current algebra treatment. Firstly, it has nonzero t channel isoscalar scattering, and secondly it has nonzero s channel P and D waves. The reason why ρ-dominance gives this extra information is that it depends on two independent parameters (the resonance mass and coupling), while the current algebra/PCAC formalism has only one (the charged pion decay amplitude F_π). There is a point of contact between the two approaches, however, through the KSRF relation (Kawarabayashi and Suzuki 1966, Riazuddin and Fayyazuddin 1966).

The last section includes a comparison between the present model and the more ambitious descriptions of the πK interaction which use crossing-symmetric Regge pole formulae. It is suggested that the scattering length predictions of these models, in both dual and interference form, are such as to indicate the need for significant satellite terms, probably containing a more realistic Pomeranchon singularity.

2. Model and predictions

The s channel isospin amplitudes for $\pi\mathrm{K} \to \pi\mathrm{K}$ are written as $A^{1/2}$ and $A^{3/2}$, and the t channel isoscalar and isovector amplitudes are denoted $A^{(+)}$ and $A^{(-)}$ to emphasize their respective even and odd symmetries under s–u crossing. They are related by

$$\sqrt{3}\, A^{(+)} = \sqrt{2}(A^{1/2}+2A^{3/2})$$
$$3A^{(-)} = 2(A^{1/2}-A^{3/2}).$$

The conventional way to express the assumption that the isospin-3/2 channel contains no resonances is to suppose that the dependence of $A^{3/2}$ on s as an independent variable may be neglected in comparison with its dependence on t and u (Yahil 1969). Then if we write

$$A^{3/2}(stu) = H(tu)$$

it follows from crossing and the symmetries of the t channel amplitudes that

$$2A^{1/2}(stu) = 3H(ts)-H(tu).$$

If either the initial or final pion has zero mass, the three invariants have values

$$s = u = M^2 \qquad t = \mu^2$$

(where M, μ denote respectively the kaon and pion mass), so that to impose the Adler condition we must arrange that H vanishes at this point. We split H into even and odd parts

$$H = F+G$$

where

$$F(tu) = F(ut)$$
$$G(tu) = -G(ut)$$

and then in the region below the physical thresholds it is possible to write convergent expansions

$$F(tu) = \sum_{r=1}^{\infty} a_r(\sigma-t-u)^r$$

$$G(tu) = (t-u) \sum_{r=1}^{\infty} b_r(\sigma-t-u)^r$$

where $\sigma = M^2+\mu^2$, and the coefficients a_r and b_r are real.

Our approximation of smoothness and simplicity prompts us to truncate both these series, and take for the amplitudes

$$F(tu) = a(\sigma-t-u)$$

and

$$G(tu) = b(\sigma-t-u)(t-u)$$

on the mass shell, where $s+t+u = 2\sigma$. This leads to a two parameter S-, P- and D-wave description of $\pi K \rightarrow \pi K$ which should be valid in the region near threshold, provided the unitarity branch points are weak.

The constants a and b appearing above can be fixed in terms of the mass (m) of the t channel ρ-meson bound state, and the product (g) of its $\pi\pi$ and $K\bar{K}$ couplings if the pole dominates the $\pi\pi \rightarrow K\bar{K}$ isovector process. That is, if

$$A^{(-)} = g\frac{s-u}{m^2-t}$$

for $t \simeq 0$, then by expanding the denominator and matching coefficients, it follows that

$$\begin{aligned} -a+b\sigma &= g/m^2 \\ b &= -g/m^4. \end{aligned}$$

Numerical values can be obtained if ρ-coupling is universal (Sakurai 1960), for then by comparison with the amplitude for $\pi\pi \rightarrow \rho \rightarrow \pi\pi$, namely

$$\frac{1}{2}\left(\frac{\gamma_\rho^{\,2}}{4\pi}\right)\frac{s-u}{m^2-t}$$

we have (at $t = 0$)

$$g = \frac{\mu}{2M}\left(\frac{\gamma_\rho^{\,2}}{4\pi}\right).$$

From the decay-rate formula

$$\Gamma = \frac{2}{3}\frac{k^3}{m^2}\left(\frac{\gamma_\rho^{\,2}}{4\pi}\right)$$

where $k = \frac{1}{2}(m^2-4\mu^2)^{1/2}$, it follows that for a $\rho \rightarrow 2\pi$ width $\Gamma = 120$ MeV we have $(\gamma_\rho^{\,2}/4\pi) = 2{\cdot}3$, and thus $g = 0{\cdot}32$. Using $m^2 = 30\mu^2$, values for a and b follow.

The interesting predictions include the values of the two S-wave scattering lengths a_1 and a_3, which we define as the values of $A^{1/2}$ and $A^{3/2}$ at threshold. The combinations corresponding respectively to isovector and isoscalar t channel scattering are

$$a_1-a_3 = 6\mu M(-a+b\sigma)$$

and

$$a_1+2a_3 = 12b(\mu M)^2$$

and ρ-dominance shows that these are respectively positive and negative, with a_1+2a_3 small. The numerical values are

$$\begin{aligned} a_1-a_3 &= 0{\cdot}23 \qquad (0{\cdot}18) \\ a_1+2a_3 &= -0{\cdot}05 \qquad (-0{\cdot}04) \end{aligned}$$

where the bracketed numbers are those obtained after division by the reduced-mass factor $(1+\mu/M)$. Table 1 gives the corresponding predictions of threshold cross sections for physical πK processes. The only experimental number available at present for comparison is the upper limit of 2–3 mb for $\pi^-K^- \to \pi^-K^-$, given by Schlein (unpublished)†.

Table 1. Predicted threshold cross sections for physical πK processes, in millibarns

$K^+\pi^+ \to K^+\pi^+$	2·1
$K^+\pi^- \to K^+\pi^-$	0·8
$K^+\pi^- \to K^0\pi^0$	2·8

The values for the scattering lengths are similar to the current algebra predictions (Weinberg 1966). The connection is evident if we use ρ-dominance and universality to write

$$a_1 - a_3 = 3\frac{\mu^2}{m^2}\left(\frac{\gamma_\rho{}^2}{4\pi}\right)$$

when consistency with the current algebra expression

$$a_1 - a_3 = 3\frac{\mu^2}{8\pi F_\pi{}^2}$$

is obtained through the KSRF relation:

$$m^2 = 2\gamma_\rho{}^2 F_\pi{}^2.$$

An important improvement which is obtained from ρ-dominance is the prediction of a definite value for a_1+2a_3. In Weinberg's calculation this combination of scattering lengths is not determined exactly, but it is usually assumed to be zero because it is proportional to the sigma term. Here it is found to be indeed small compared to a_1-a_3, and, in addition, to be negative in sign. This agrees with the prediction

$$a_1+2a_3 < 0$$

which we made previously (Johnson 1970) on general grounds of analyticity and unitarity, given the existence of the K*(890) resonance.

The P- and D-wave scattering lengths are found to be positive in the $I = \frac{1}{2}$ channel and smaller and negative in $I = \frac{3}{2}$, as might be expected. The size of the isospin-1/2 P-wave scattering length is however only 25% of the value given by the tail of the K*(890) pole, which suggests that this narrow resonance does not dominate right down to threshold, as the broader ρ-meson does in $\pi\pi$ scattering.

3. Conclusions

An encouraging result of this theory of the πK threshold interaction is the determination of a small negative value of the scattering-length combination a_1+2a_3. Other definite predictions for this quantity are given by the currently fashionable crossing-symmetric Regge pole models. In existing calculations both the dual formula (Lovelace unpublished‡, Kawarabayashi *et al.* 1969, Arnowitt *et al.* 1969)

† Invited talk at the Argonne–Purdue Conference (see footnote, p. 883).
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and the interference model (Curry *et al.* 1971) give on the contrary $a_1+2a_3 > 0$. This disagrees with general arguments (Johnson 1970), and suggests that the Regge pole models may need extra (satellite) terms. In the dual model in particular, Pomeranchon exchange is ignored, and since this contributes to the crossing-even amplitude it is most probably the source of the discrepancy.

Because the threshold branch points are ignored, we cannot give detailed phase shift predictions at present. However some indications can be obtained by interpreting our polynomial partial wave amplitudes as K-matrix elements. This is a good approximation for small phase shifts, and gives the results summarized in table 2 for centre of mass energy, $\sqrt{s}$ = 890 MeV (under the K* resonance). A full treatment of the left-hand cuts is necessary to investigate the possibility of a kappa resonance.

Table 2. Phase shifts at $\sqrt{s}$ = 890 MeV, calculated as described in the text

$S_{1/2}$	12°
$D_{1/2}$	2°
$S_{3/2}$	−18°
$P_{3/2}$	−1°
$D_{3/2}$	−1°

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